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Mark Scheme (Results)

June 2011

GCE Further Pure FP1 (6667) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

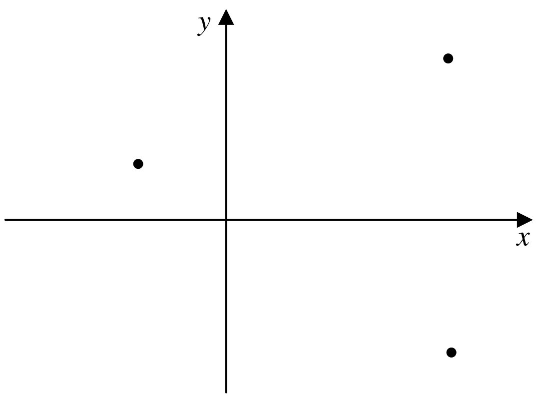
3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper
   - $\sqcup$ The second mark is dependent on gaining the first mark

June 2011
6667 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = 3^x + 3x - 7$		
(a)	$\mathrm{f}(1) = -1$ $\mathrm{f}(2) = 8$	Either any one of f(1) = -1 or f(2) = 8.	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x = 1$ and $x = 2$.	Both values correct, sign change and conclusion	A1
			(2)
(b)	$\mathrm{f}(1.5) = 2.696152423...$ $\{\Rightarrow 1 \leq \alpha \leq 1.5\}$	$\mathrm{f}(1.5) =$ awrt 2.7 (or truncated to 2.6)	B1
	$\mathrm{f}(1.25) = 0.698222038...$ $\Rightarrow 1 \leq \alpha \leq 1.25$	Attempt to find $\mathrm{f}(1.25)$.	M1
		$\mathrm{f}(1.25) =$ awrt 0.7 with $1 \leq \alpha \leq 1.25$ or $1 < \alpha < 1.25$ or $[1, 1.25]$ or $(1, 1.25)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		(3)
			5

Question Number	Scheme	Notes	Marks
2. (a)	$\lvert z_1 \rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	$\sqrt{5}$ or awrt 2.24	B1
			(1)
(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	$\tan^{-1}\left(\frac{1}{2}\right)$ or $\tan^{-1}\left(\frac{2}{1}\right)$ or $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	M1
	$= 2.677945045... = 2.68$ (2 dp)	awrt 2.68	A1 oe
	Can work in degrees for the method mark ($\arg z = 153.4349488^{\circ}$)		(2)
	$\arg z = \tan^{-1}\left(\frac{1}{-2}\right) = -0.46$ on its own is M0 but $\pi + \tan^{-1}\left(\frac{1}{-2}\right) = 2.68$ scores M1A1 $\pi - \tan^{-1}\left(\frac{1}{-2}\right) =$ is M0 as is $\pi - \tan\left(\frac{1}{2}\right)$ (2.60)		
(c)	$z^2 - 10z + 28 = 0$		
	$z = \dfrac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	An attempt to use the quadratic formula (usual rules)	M1
	$= \dfrac{10 \pm \sqrt{100 - 112}}{2}$		
	$= \dfrac{10 \pm \sqrt{-12}}{2}$		
	$= \dfrac{10 \pm 2\sqrt{3}\,\mathrm{i}}{2}$	Attempt to simplify their $\sqrt{-12}$ in terms of i. E.g. $\mathrm{i}\sqrt{12}$ or $\mathrm{i}\sqrt{3 \times 4}$	M1
	If their b^2-4ac >0 then only the first M1 is available.		
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\,\mathrm{i}$	A1 oe
	Correct answers with no working scores full marks. **See appendix for alternative solution by completing the square**		(3)
(d)		Note that the points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$.	
		The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label.	B1
		The distinct points z_2 and z_3 plotted correctly and symmetrically about the x-axis on the Argand diagram with/without label.	B1 $\sqrt{}$
	The points must be correctly placed relative to each other. If you are in doubt about awarding the marks then consult your team leader or use review.		(2)
	NB the second B mark in (d) depends on having obtained complex numbers in (c)		
			8

Question Number	Scheme	Notes	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 1+2 & \sqrt{2}-\sqrt{2} \\ \sqrt{2}-\sqrt{2} & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1
			(2)
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**;	B1;
		scale factor **3**, centre **(0, 0)**	B1
	Allow 'from' or 'about' for centre and 'O' or 'origin' for (0, 0)		
			(2)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		
	Reflection; in the line $y = -x$.	**Reflection**;	B1;
		$y = -x$	B1
	Allow 'in the axis' 'about the line' $y = -x$ etc.		
	The question does not specify a <u>single</u> transformation so we would need to accept any combinations that are correct e.g. Anticlockwise rotation of 90^{o} about the origin followed by a reflection in the x-axis is acceptable. In cases like these, the combination has to be <u>completely</u> correct and scored as B2 (no part marks). If in doubt consult your Team Leader.		(2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.		
	$\mathbf{C}$ is singular $\Rightarrow \det \mathbf{C} = 0$. (Can be implied)	$\det \mathbf{C} = 0$	B1
	Special Case $\frac{1}{9(k+1)-12k} = 0$ **B1(implied)M0A0**		
	$9(k+1) - 12k\ (= 0)$	Applies $9(k+1) - 12k$	M1
	$9k + 9 = 12k$		
	$9 = 3k$		
	$k = 3$	$k = 3$	A1
	k = 3 with no working can score full marks		(3)
			9

Question Number	Scheme	Notes	Marks
4.	$\mathrm{f}(x) = x^2 + \dfrac{5}{2x} - 3x - 1\,, \quad x \neq 0$		
(a)	$\mathrm{f}(x) = x^2 + \dfrac{5}{2}x^{-1} - 3x - 1$		
	$\mathrm{f}'(x) = 2x - \dfrac{5}{2}x^{-2} - 3\,\{+\,0\}$	At least two of the four terms differentiated correctly.	M1
		Correct differentiation. (Allow any correct unsimplified form)	A1
	$\left\{\mathrm{f}'(x) = 2x - \dfrac{5}{2x^2} - 3\right\}$		(2)
(b)	$\mathrm{f}(0.8) = 0.8^2 + \dfrac{5}{2(0.8)} - 3(0.8) - 1 (= 0.365)\left(= \dfrac{73}{200}\right)$	A correct numerical expression for f(0.8)	B1
	$\mathrm{f}'(0.8) = -5.30625\left(= \dfrac{-849}{160}\right)$	Attempt to insert $x = 0.8$ into their f'(x). Does not require an evaluation. (If $\mathrm{f}'(0.8)$ is incorrect for their derivative and there is no working score M0)	M1
	$\alpha_2 = 0.8 - \left(\dfrac{"0.365"}{"-5.30625"}\right)$	Correct application of Newton-Raphson using their values. Does not require an evaluation.	M1
	$= 0868786808...$		
	$= 0.869\ \ (3\text{dp})$	0.869	A1 **cao**
	A correct answer only with no working scores no marks. N-R must be seen. Ignore any further applications of N-R		(4)
	A derivative of $2x - 5(2x)^{-2} - 3$ is quite common and leads to f'(0.8) = -3.353125 and a final answer of 0.909. This would normally score M1A0B1M1M1A0 (4/6) Similarly for a derivative of $2x - 10x^{-2} - 3$ where the corresponding values are f'(0.8) = -17.025 and answer 0.821		
			6

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>5.</td><td>$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.</td><td></td><td></td></tr>
<tr><td>(a)</td><td>$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$</td><td></td><td></td></tr>
<tr><td></td><td>Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$</td><td>Using the information in the question to form the matrix equation. Can be implied by both correct equations below.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">Do not allow this mark for other incorrect statements unless interpreted correctly later
e.g. $\begin{pmatrix} 4 \\ 6 \end{pmatrix}\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$ would be M0 unless followed by correct equations or $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$</td><td></td></tr>
<tr><td></td><td>So, $-16 + 6a = 2$ and $4b - 12 = -8$
Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$</td><td>Any one correct equation.
Any correct horizontal line</td><td>M1</td></tr>
<tr><td></td><td rowspan="2">giving $a = 3$ and $b = 1$.</td><td>Any one of $a = 3$ or $b = 1$.</td><td>A1</td></tr>
<tr><td></td><td>Both $a = 3$ and $b = 1$.</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(4)</td></tr>
<tr><td rowspan="2">(b)</td><td rowspan="2">$\det \mathbf{A} = 8 - (3)(1) = 5$</td><td>Finds determinant by applying $8 -$ their ab.</td><td>M1</td></tr>
<tr><td>$\det \mathbf{A} = 5$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Special case: The equations -16 + 6b = 2 and 4a – 12 = -8 give a = 1 and b = 3. This comes from incorrect matrix multiplication. This will score nothing in (a) but allow all the marks in (b).</td><td></td></tr>
<tr><td></td><td colspan="2">Note that $\det \mathbf{A} = \frac{1}{8-ab}$ scores M0 here but the following 2 marks are available. However, beware $\det \mathbf{A} = \frac{1}{8-ab} = \frac{1}{5} \Rightarrow area\, S = \frac{30}{\frac{1}{5}} = 150$
This scores M0A0 M1A0</td><td></td></tr>
<tr><td></td><td>Area $S = (\det \mathbf{A})(\text{Area } R)$</td><td></td><td></td></tr>
<tr><td></td><td rowspan="2">Area $S = 5 \times 30 = 150 \text{ (units)}^2$</td><td>$\frac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$</td><td>M1</td></tr>
<tr><td></td><td>150 or ft answer</td><td>A1 $\sqrt{}$</td></tr>
<tr><td></td><td colspan="2">If their det A < 0 then allow ft provided final answer > 0</td><td>(4)</td></tr>
<tr><td></td><td colspan="2">In (b) Candidates may take a more laborious route for the area scale factor and find the area of the unit square, for example, after the transformation represented by A. This needs to be a complete method to score any marks. Correctly establishing the area scale factor M1. Correct answer 5 A1. Then mark as original scheme.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>8</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6.	$z + 3\mathrm{i}\,z^* = -1 + 13\mathrm{i}$		
	$(x + \mathrm{i}\,y) + 3\mathrm{i}(x - \mathrm{i}\,y)$	$z^* = x - \mathrm{i}\,y$	B1
		Substituting $z = x + \mathrm{i}\,y$ and their z^* into $z + 3\mathrm{i}\,z^*$	M1
	$x + \mathrm{i}\,y + 3\mathrm{i}\,x + 3\,y = -1 + 13\mathrm{i}$	Correct equation in x and y with $\mathrm{i}^2 = -1$. Can be implied.	A1
	$(x + 3\,y) + \mathrm{i}(y + 3x) = -1 + 13\mathrm{i}$		
	Re part: $x + 3y = -1$ Im part: $y + 3x = 13$	An attempt to equate real **and** imaginary parts.	M1
		Correct equations.	A1
	$3x + 9y = -3$ $3x + y = 13$		
	$8y = -16 \Rightarrow y = -2$	Attempt to solve simultaneous equations to find one of x or y. **At least one of the equations must contain both x and y terms.**	M1
	$x + 3y = -1 \Rightarrow x - 6 = -1 \Rightarrow x = 5$	Both $x = 5$ and $y = -2$.	A1
	$\{z = 5 - 2\mathrm{i}\}$		(7)
			7

Question Number	Scheme	Notes	Marks
7.	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= \underline{4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1) + n}$	First two terms correct.	A1
		$+ n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
	Note that there are no marks for proof by induction.		
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	Note that (b) says **hence** so they have to be using the result from (a)		
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2-1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		(4)
			10

Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 48x$ with general point $P\left(12t^2, 24t\right)$.		
(a)	$y^2 = 4ax \Rightarrow \quad a = \frac{48}{4} = 12$	Using $y^2 = 4ax$ to find a.	M1
	So, directrix has the equation $x + 12 = 0$	$x + 12 = 0$	A1 oe
	Correct answer with no working allow full marks		(2)
(b)	$y = \sqrt{48}\, x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\sqrt{48}\, x^{-\frac{1}{2}} \left(= 2\sqrt{3}\, x^{-\frac{1}{2}}\right)$ or (implicitly) $y^2 = 48x \Rightarrow 2y\frac{dy}{dx} = 48$ or (chain rule) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 24 \times \frac{1}{24t}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\, x^{-\frac{1}{2}}$ $ky\frac{\mathrm{d}y}{\mathrm{d}x} = c$ their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	M1
	When $x = 12t^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{48}}{2\sqrt{12t^2}} = \frac{\sqrt{4}}{2t} = \frac{1}{t}$ or $\frac{dy}{dx} = \frac{48}{2y} = \frac{48}{48t} = \frac{1}{t}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$	A1
	T: $y - 24t = \frac{1}{t}\left(x - 12t^2\right)$	Applies $y - 24t =$ their $m_T\left(x - 12t^2\right)$ or $y = \left(\text{their } m_T\right)x + c$ using $x = 12t^2$ *and* $y = 24t$ in an attempt to find c. **Their m_T must be a function of t .**	M1
	T: $ty - 24t^2 = x - 12t^2$		
	T: $x - ty + 12t^2 = 0$	Correct solution.	A1 **cso** *
	Special case: If the gradient is <u>quoted</u> as 1/t, this can score M0A0M1A1		(4)
(c)	Compare $P\left(12t^2, 24t\right)$ with $(3, 12)$ gives $t = \frac{1}{2}$.	$t = \frac{1}{2}$	B1
	NB $x - ty + 12t^2 = 0$ with $x = 3$ and $y = 12$ gives $4t^2 - 4t + 1 = 0 = \left(2t - 1\right)^2 \Rightarrow t = \frac{1}{2}$		
	$t = \frac{1}{2}$ into **T** gives $x - \frac{1}{2}y + 3 = 0$	Substitutes their t into **T**.	M1
	See Appendix for an alternative approach to find the tangent		
	At X, $x = -12 \Rightarrow -12 - \frac{1}{2}y + 3 = 0$	Substitutes their x from (a) into **T**.	M1
	So, $-9 = \frac{1}{2}y \Rightarrow y = -18$		
	So the coordinates of X are $\left(-12, -18\right)$.	$\left(-12, -18\right)$	A1
	The coordinates must be together at the end for the final A1 e.g. as above or x = -12, y = -18		(4)
			10

Question Number	Scheme	Notes	Marks
9. (a)	$n=1$; LHS $=\begin{pmatrix}3&0\\6&1\end{pmatrix}^1=\begin{pmatrix}3&0\\6&1\end{pmatrix}$ RHS $=\begin{pmatrix}3^1&0\\3(3^1-1)&1\end{pmatrix}=\begin{pmatrix}3&0\\6&1\end{pmatrix}$ As LHS = RHS, the matrix result is true for $n=1$.	Check to see that the result is true for $n=1$.	B1
	Assume that the matrix equation is true for $n=k$, ie. $\begin{pmatrix}3&0\\6&1\end{pmatrix}^k=\begin{pmatrix}3^k&0\\3(3^k-1)&1\end{pmatrix}$		
	With $n=k+1$ the matrix equation becomes $\begin{pmatrix}3&0\\6&1\end{pmatrix}^{k+1}=\begin{pmatrix}3&0\\6&1\end{pmatrix}^k\begin{pmatrix}3&0\\6&1\end{pmatrix}$		
	$=\begin{pmatrix}3^k&0\\3(3^k-1)&1\end{pmatrix}\begin{pmatrix}3&0\\6&1\end{pmatrix}$ or $\begin{pmatrix}3&0\\6&1\end{pmatrix}\begin{pmatrix}3^k&0\\3(3^k-1)&1\end{pmatrix}$	$\begin{pmatrix}3^k&0\\3(3^k-1)&1\end{pmatrix}$ by $\begin{pmatrix}3&0\\6&1\end{pmatrix}$	M1
	$=\begin{pmatrix}3^{k+1}+0&0+0\\9(3^k-1)+6&0+1\end{pmatrix}$ or $\begin{pmatrix}3^{k+1}+0&0+0\\6.3^k+3(3^k-1)&0+1\end{pmatrix}$	Correct unsimplified matrix with no errors seen.	A1
	$=\begin{pmatrix}3^{k+1}&0\\9(3^k)-3&1\end{pmatrix}$		
	$=\begin{pmatrix}3^{k+1}&0\\3\left(3(3^k)-1\right)&1\end{pmatrix}$		
	$=\begin{pmatrix}3^{k+1}&0\\3(3^{k+1}-1)&1\end{pmatrix}$	Manipulates so that $k\to k+1$ on at least one term.	dM1
		Correct result with no errors seen with some working between this and the previous A1	A1
	If the result is true for $n=k$, (1) then it is now true for $n=k+1$. (2) As the result has shown to be true for $n=1$, (3) then the result is true for all n. (4) **All 4 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with all previous marks earned	A1 cso
			(6)

Question Number	Scheme	Notes	Marks
9. (b)	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. $\{\therefore f(n)$ is divisible by 12 when $n = 1.\}$		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct unsimplified expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$\therefore f(k+1) - f(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	Applies $f(k+1) - f(k)$. No simplification is necessary and condone missing brackets.	M1
	$= 7^{2k+1} - 7^{2k-1}$		
	$= 7^{2k-1}\left(7^2 - 1\right)$	Attempting to isolate 7^{2k-1}	M1
	$= 48\left(7^{2k-1}\right)$	$48\left(7^{2k-1}\right)$	A1cso
	$\therefore f(k+1) = f(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $f(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$,(4) then the result is true for all n. (5). **All 5 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with no incorrect work. Don't condone missing brackets.	A1 cso
	There are other ways of proving this by induction. See appendix for 3 alternatives. If you are in any doubt consult your team leader and/or use the review system.		(6)
			12

Appendix

Question Number	Scheme	Notes	Marks
Aliter **2.** (c) **Way 2**			
	$z^2 - 10z + 28 = 0$		
	$(z-5)^2 - 25 + 28 = 0$	$(z \pm 5)^2 \pm 25 + 28 = 0$	M1
	$(z-5)^2 = -3$		
	$z - 5 = \sqrt{-3}$		
	$z - 5 = \sqrt{3}\,\mathrm{i}$	Attempt to express their $\sqrt{-3}$ in terms of i.	M1
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\,\mathrm{i}$	A1 oe
			(3)

Question Number	Scheme		Marks
Aliter **2.** (c) **Way 3**			
	$z^2 - 10z + 28 = 0$		
	$\left(z-\left(p+i\sqrt{q}\right)\right)\left(z-\left(p-i\sqrt{q}\right)\right) = z^2 - 2pz + p^2 + q$		
	$2p = \pm 10$ *and* $p^2 \pm q = 28$	Uses sum and product of roots	M1
	$2p = \pm 10 \Rightarrow p = 5$	Attempt to solve for p(or q)	M1
	$p = 5$ *and* $q = 3$		A1
			(3)

Question Number	Scheme	Notes	Marks
Aliter **8.** (c) **Way 2**	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}\,x^{-\frac{1}{2}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$		B1
	Gives $y - 12 = 2(x - 3)$	Uses (3, 12) and their "2" to find the equation of the tangent.	M1
	$x = -12 \Rightarrow y - 12 = 2(-12 - 3)$	Substitutes their x from (a) into their tangent	M1
	$y = -18$		
	So the coordinates of X are $(-12, -18)$.		A1
			(4)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 2**	$\mathrm{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12,$	Shows that $\mathrm{f}(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f (n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$,		
	$\mathrm{f}(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	So, $\mathrm{f}(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $\mathrm{f}(k+1)$.	B1
	giving, $\mathrm{f}(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (7^{2k-1} + 5) - 240$	M1 Attempt to isolate $7^{2k-1} + 5$	M1
	$\mathrm{f}(k+1) = 49 \times \mathrm{f}(k) - 240$	Correct expression in terms of f(k)	A1
	As both f(k) and 240 are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 3**			
	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f(n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, f(k) is divisible by 12		
	so $f(k) = 7^{2k-1} + 5 = 12m$		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 7^2.7^{2k-1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (12m - 5) + 5$	Substitute for m	M1
	$f(k+1) = 49 \times 12m - 240$	Correct expression in terms of m	A1
	As both $49 \times 12m$ and 240 are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 4**			
	$\mathrm{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $\mathrm{f}(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ $\mathrm{f}(n)$ is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $\mathrm{f}(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	$\mathrm{f}(k+1) + 35\mathrm{f}(k) = \underline{7^{2(k+1)-1} + 5} + 35(7^{2k-1} + 5)$	Correct expression for $\mathrm{f}(k+1)$.	B1
	$\mathrm{f}(k+1) + 35\mathrm{f}(k) = 7^{2k+1} + 5 + 35(7^{2k-1} + 5)$	Add appropriate multiple of f(k) For 7^{2k} this is likely to be 35 (119, 203,.) For 7^{2k-1} 11 (23, 35, 47,..)	M1
	giving, $7.7^{2k} + 5 + 5.7^{2k} + 175$	Attempt to isolate 7^{2k}	M1
	$= 180 + 12 \times 7^{2k} = 12\left(15 + 7^{2k}\right)$	Correct expression	A1
	$\therefore \mathrm{f}(k+1) = 12\left(7^{2k} + 15\right) - 35\mathrm{f}(k)$. As both f($k$) and $12\left(7^{2k} + 15\right)$ are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

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